

NEW ACCEPTANCE SAMPLING PLANS BASED ON TRUNCATED LIFE TESTS FOR

KOMAL DISTRIBUTION

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ABSTRACT

When it is impractical to examine every item in a production batch, acceptance sampling methods are used to improve and control quality. This process relies on a random sample to determine whether to accept or reject the entire batch. Variations in the lifetime distribution of the sample are considered, as they may differ across the batches. In this research paper, we investigate a new lifetime distribution, the Komal distribution, to develop a distinctive single-acceptance sampling plan. In the development of a new acceptance sampling plan, it is essential to account for the constrained timeframe available for conducting the mean lifetime evaluation. This consideration is instrumental in determining crucial plan parameters, such as the operating characteristics function and the minimum requisite sample size. Based on these factors, the risk incurred by the producer for the entire material batch can be calculated.

KEYWORDS: Acceptance Sampling Plan; Consumer's Risk, Operating Characteristic Function, Komal Distribution, Producer's Risk, Truncated Life Tests

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1. INTRODUCTION

Single acceptance sampling plans play a crucial role in the production sector, as they enable the assessment of a lot's acceptability based on its lifetime. Manufacturers prioritize producing high-quality products at lower costs, while customers seek durable, high-quality goods. Notably, high-quality products typically have a higher acceptance rate compared to low-quality ones. By examining a small sample from the entire batch, quality can be evaluated according to the acceptance sampling plan protocol. The acceptance sampling plan specifies the sampling policies and standards for accepting or rejecting a lot. This strategy can be applied to test the initial materials used in production processes or the finished goods. There are two main types of acceptance sampling plans: attribute-based and variable-based.

The single acceptance sampling plan has been the subject of investigation by various researchers in the context of truncated lifetime tests, based on the presumption that a product's lifetime conforms to a specific probability distribution. Rao et al. (2012) developed acceptance sampling plans based on truncated life tests for the inverse Rayleigh distribution, and Al-Nasser and Al-Omari (2013) proposed similar plans for the exponentiated Fréchet distribution. Gui and Zhang (2014) examined a life test based on the Gompertz distribution, and Al-Omari (2014) introduced a three-parameter Kappa distribution. Al -Omari (2016) proposed a life test for transmuted inverse Rayleigh distribution; Braimah et al. (2016) investigated single truncated acceptance sampling plans for Weibull product life distributions. Malathi and Muthulakshmi

(2017) proposed an economic design of acceptance sampling plans for truncated life test using Fréchet distribution. Mahdy et al. (2018) considered skew-generalized inverse Weibull distribution in acceptance sampling. Al-Omari et al. (2018) proposed acceptance sampling plans based on truncated life tests for Sushila distribution.

Al-Nasser et al. (2018) developing single acceptance sampling plans based on truncated lifetime test for an Ishita distribution. Al-Omari and Al-Nasser (2019) introduced new acceptance sampling plans for two parameter quasi Lindley distribution. Al-Omari et al. (2019) proposed acceptance sampling plans based on truncated life tests three-parameter Lindley distribution. Al-Omari et al. (2019) proposed acceptance sampling plans for Rama distribution. Al-Nasser and Obeidat (2020) studied acceptance sampling plans for Tsallis q-exponential distribution. Al-Omari et al. (2020) developed a new acceptance plans for Akash distribution with an application to electric carts data. Al-Omari et al. (2021) proposed acceptance sampling plans under two- parameter Quasi Shanker distribution.

This manuscript is structured as follows: Section 2 introduces the Komal distribution and examines its statistical properties. Subsequently, Section 3 presents novel sampling plans grounded in the Komal distribution, including analyses of the minimum sample size, operating characteristic function, and producer's risk. Relevant tabular data and illustrative examples are provided in Section 4. An application utilizing real-world empirical data is then offered in Section 5. Lastly, the principal conclusions are summarized in the final Section 6.

2. KOMAL DISTRIBUTION

Rama Shanker (2023) developed Komal distribution with probability density function (PDF) defined as

$$f(x,\theta) = \frac{\theta^2}{\theta^2 + \theta + 1} (1 + \theta + x) e^{-\theta x} \qquad ; x > 0, \ \theta > 0$$
⁽¹⁾

The corresponding cumulative distribution function (CDF) of Komal distribution is given by

$$F(x,\theta) = 1 - \left[1 + \frac{\theta x}{\theta^2 + \theta + 1}\right]e^{-\theta x} \qquad ; x > 0, \ \theta > 0$$
⁽²⁾

The r^{th} moment about origin of the Komal random variable can be obtained by

$$\mu_{r}^{\dagger} = \frac{r! \left[\theta^{2} + \theta + r + 1\right]}{\theta^{r} \left(\theta^{2} + \theta + 1\right)} ; r = 1, 2, 3, \dots$$

Thus the first and second moments are

$$\mu_{1}^{\dagger} = \frac{\theta^{2} + \theta + 2}{\theta(\theta^{2} + \theta + 1)} \quad \text{and} \quad \mu_{2}^{\dagger} = \frac{2(\theta^{2} + \theta + 3)}{\theta^{2}(\theta^{2} + \theta + 1)}$$
(3)

And then the variance of Komal distribution is

$$\mu_{2} = \frac{\theta^{4} + 2\theta^{3} + 5\theta^{2} + 4\theta + 2}{\theta^{2} \left(\theta^{2} + \theta + 1\right)^{2}}$$

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The coefficient of variation (C.V.) coefficient of skewness $(\sqrt{\beta_1})$ coefficient of kurtosis (β_2) of Komal distribution are obtained as

$$C.V. = \frac{\sqrt{\mu_2}}{\mu_1^1} = \frac{\sqrt{\theta^4 + 2\theta^3 + 5\theta^2 + 4\theta + 2}}{\theta^2 + \theta + 2}$$
$$\sqrt{\beta_1} = \frac{\mu_3}{\mu_2^{3/2}} = \frac{4(\theta^6 + 3\theta^5 + 9\theta^4 + 13\theta^3 + 12\theta^2 + 6\theta + 2)}{(\theta^4 + 2\theta^3 + 5\theta^2 + 4\theta + 2)^{3/2}}$$
$$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{3(3\theta^8 + 12\theta^7 + 434\theta^6 + 1260\theta^5 + 2471\theta^4 + 2856\theta^3 + 2428\theta^2 + 1208\theta + 400)}{(\theta^4 + 2\theta^3 + 5\theta^2 + 4\theta + 2)^2}$$

Let be a continuous random variable with pdf f(x) and F(x). The hazard rate function and the mean residual life function of X are defined as

$$h(x) = \frac{f(x)}{1 - F(x)}$$
$$m(x) = E(X - x \mid X > x) = \frac{1}{1 - F(x)} \int_{x}^{\infty} [1 - F(t)] dt$$

The corresponding hazard rate function, h(x) and the mean residual life function, m(x) of Komal distribution are given by

$$h(x) = \frac{\theta^2 (1 + \theta + x)}{(\theta^2 + \theta + 1 + \theta x)}$$
$$m(x) = \frac{\theta^2 + \theta + 2 + \theta x}{\theta(\theta^2 + \theta + 1 + \theta x)}$$

The maximum likelihood estimator of θ is given by solving the equation

$$\frac{n\left(\theta+2\right)}{\theta\left(\theta^{2}+\theta+1\right)} + \sum_{i=1}^{n} \frac{1}{1+\theta+x_{i}} - n\overline{x} = 0$$

$$\tag{4}$$

Where \overline{x} is the sample mean.

3. DESIGN OF THE ACCEPTANCE SAMPLING PLAN

Assume that life time of the product follow the Komal distribution defined in equation (1). Let the life test terminates at a predetermined time t_0 and the number of failures within this time interval [0, t] are obtained. The lot occurs is accepted if the number of failures at the time t_0 does not exceed the acceptance number c.

An acceptance sampling plan based on truncated life tests consist of the following quantities

- i. The number of units (n) on test.
- ii. An acceptance number (c) where if c or less failures happened the test time (t), the lot is acceptable.
- iii. The maximum test duration time, t.
- iv. iv)The ratio t/μ_0 where μ_0 is the specified average life

The researchers assume an infinitely large lot size in order to utilize the theory of binomial distribution during the experiment. Assume that the consumer's risk (the probability of acceptance a bad lot) is determined to be at most $1 - P^*$ that is the probability that the real mean life μ is less than μ_0 , not exceed $1 - P^*$. Our problem is to get the smallest sample size *n* necessary to satisfy the inequality.

3.1 Minimum Sample Size

Here we assume that the lot size is infinitely large so that the theory of binomial distribution can be applied. Assume that the consumer's risk (the probability that a bad lot is accepted) is determined to be at most $1 - P^*$ i.e. the probability that the real mean life μ is less than μ_0 not exceeds $1 - P^*$. Here, the problem is to determine the smallest sample size *n* necessary to satisfies the inequality

$$\sum_{i=0}^{c} \binom{n}{i} p^{i} (1-p)^{n-i} \le 1-P^{*}$$
(5)

Where *c* is the acceptance number for given values of $P^* \in (0,1)$ where $p = F(t,\mu_0)$ is the probability of a failure within the time *t* which depends only on the ratio t/μ_0 . If the number of observed failures within the time *t* is at most *c* then from equation (5) we can confirm with probability *P* that $F(t,\mu) \leq F(t,\mu_0)$ which implies $\mu_0 \leq \mu$. The minimum sample size values satisfying (5) have been calculated for $P^* = 0.75, 0.90, 0.95, 0.99$ and $t/\mu_0 = 0.628, 0.942, 1.257, 1.571, 2.356, 3.141, 3.927, 4.712$. The values of t/μ_0 and P^* are consist of corresponding values Baklizi and El Masri (2004) and Kantam et al. (2001).

3.2 Operating Characteristic of Sampling Plan $(n, c, t/\mu_0)$

The operating characteristic function of the sampling plan $(n, c, t/\mu_0)$ is the probability of accepting the lot. It can be considered as a source for choosing for choosing the minimum sample size *n*, or the acceptance number *c*. The operating characteristic function of the suggested acceptance sampling plan is defined as

$$OC(p) = P(Accepting \ a \ lot \ | \ \mu < \mu_0) = \sum_{i=0}^{c} \binom{n}{i} p^i (1-p)^{n-i}$$
(6)

where $p = F(t, \mu_0)$

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3.3 Producer's Risk

The producer's risk (PR) is the probability of rejection of lot when it is good, that $\mu > \mu_0$. It is given by

$$PR(p) = P(\text{Rejecting a lot } | \mu > \mu_0) = \sum_{i=c+1}^n \binom{n}{i} p^i (1-p)^{n-i}$$

For a given value of the producer's risk say λ , under a given sampling plan, one may be interested in knowing what is the smallest value of μ/μ_0 is that will assert a PR of at most λ . The value of μ/μ_0 is the minimum positive

number for which
$$p = F\left(\frac{t}{\mu_0} \frac{\mu_0}{\mu}\right)$$
 satisfies the inequality

$$PR(p) = \sum_{i=c+1}^{n} \binom{n}{i} p^{i} (1-p)^{n-i} \leq \lambda$$

4. Description of Tables

Assuming that the life distribution follows the Komal distribution and Table 1 displayed the minimum sample size needed to assert that μ is greater than μ_0 with probability at least P^* with c as an acceptance number. For illustration when $P^* = 0.95$, c = 2 and $t/\mu_0 = 0.942$, the corresponding entry table is n = 8 Hence out of the 8 items, less than or equal to two fail before time t, then the decision is the lot can be accepted with a probability of 0.95. This means that out of 8 items, if there are two items fail previous time t, then a 95% upper confidence interval for μ is $(t/0.942, \infty)$ Table 2 devoted to the operating characteristic function values for the suggested acceptance sampling plan and for the plan $(n = 8, c = 2, t/\mu_0 = 0.942)$ with $P^* = 0.95$ the operating characteristic function values are

| μ/μ_0 | 2 | 4 | 6 | 8 | 10 | 12 |
|-------------|---------|---------|---------|---------|---------|---------|
| OC(p) | 0.37797 | 0.78223 | 0.90649 | 0.95228 | 0.97257 | 0.98285 |
| PR | 0.62203 | 0.21777 | 0.09351 | 0.04772 | 0.02743 | 0.01715 |

Form this operating characteristic function values, it is found that if the real mean life time is twice the identified mean life, then the producer's risk is approximately 0.62203 and approximately zero for large values of μ/μ_0 .

Table 3 includes the values of the minimum ratio of the true average life to the identified mean lifetime (μ/μ_0) for different choices of c and t/μ_0 provided that the producer's risk is 0.05. Hence, for $(n = 8, c = 2, t/\mu_0 = 0.942)$ the value of μ/μ_0 is 3.0957. This displays that the product must have a mean life of 3.0957 times the determined mean life 1000 hours accept the lot with probability of at least 0.95

Table 1. Minimum sample sizes to be tested for a time *t* assert with probability P^* and acceptance number *c* that $\mu > \mu_0$ $\mu > \mu_0$ for $\theta = 2$ in the Komal distribution.

| D* | C | t/μ_0 | | | | | | | | |
|------|---------|-----------|----------|----------|----------|----------|----------|-------|-------|--|
| P | ι | 0.628 | 0.942 | 1.257 | 1.571 | 2.356 | 3.141 | 3.927 | 4.712 | |
| | 0 | 3 | 2 | 2 | 1 | 1 | 1 | 1 | 1 | |
| | 1 | 5 | 4 | 3 | 3 | 2 | 2 | 2 | 2 | |
| | 2 | 8 | 6 | 5 | 4 | 4 | 3 | 3 | 3 | |
| | 3 | 10 | 8 | 6 | 6 | 5 | 4 | 4 | 4 | |
| | 4 | 13 | 10 | 8 | 7 | 6 | 5 | 5 | 5 | |
| 0.75 | 5 | 15 | 11 | 9 | 8 | 7 | 6 | 6 | 6 | |
| | 6 | 18 | 13 | 11 | 10 | 8 | 8 | 7 | 7 | |
| | 7 | 20 | 15 | 12 | 11 | 9 | 9 | 8 | 8 | |
| | 8 | 22 | 17 | 14 | 12 | 10 | 10 | 9 | 9 | |
| | 9 | 25 | 18 | 15 | 14 | 12 | 11 | 10 | 10 | |
| | 10 | 27 | 20 | 1/ | 15 | 13 | 12 | 11 | 11 | |
| | 1 | 4 | 5 | 2 1 | | 1 | 1 | 2 | 2 | |
| | 2 | 10 | 7 | - - | 5 | 3 4 | 2 4 | 2 | 23 | |
| | 3 | 13 | 9 | 8 | 7 | 5 | 5 | 4 | 4 | |
| | 4 | 16 | 11 | 9 | 8 | 7 | 6 | 5 | 5 | |
| 0.90 | 5 | 18 | 13 | 11 | 9 | 8 | 7 | 7 | 6 | |
| | 6 | 21 | 15 | 12 | 11 | 9 | 8 | 8 | 7 | |
| | 7 | 23 | 17 | 14 | 12 | 10 | 9 | 9 | 8 | |
| | 8 | 26 | 19 | 16 | 14 | 11 | 10 | 10 | 9 | |
| | 9 | 28 | 21 | 17 | 15 | 12 | 11 | 11 | 10 | |
| | 10 | 31 | 23 | 19 | 16 | 14 | 12 | 12 | 11 | |
| | 0 | 5 | 4 | 3 | 2 | 2 | 1 | 1 | 1 | |
| | 1 | 9 | 6 | 5 | 4 | 3 | 3 | 2 | 2 | |
| | 2 | 12 | 8 | 9 | 6 | 4 | 4 | 4 | 3 | |
| | 3 | 15 | 10 | 8 10 | / | 07 | 5 | 5 | 4 | |
| 0.05 | 4 | 20 | 15 | 10 | 9 | / 8 | 07 | 07 | 5 | |
| 0.95 | 5 | 20 | 15 | 12 | 10 | 0 | 8 | 8 | 8 | |
| | 7 | 26 | 19 | 15 | 13 | 11 | 10 | 9 | 9 | |
| | 8 | 28 | 20 | 17 | 15 | 12 | 11 | 10 | 10 | |
| | 9 | 31 | 22 | 18 | 16 | 13 | 12 | 11 | 11 | |
| | 10 | 33 | 24 | 20 | 17 | 14 | 13 | 12 | 12 | |
| | 0 | 8 | 5 | 4 | 3 | 2 | 2 | 2 | 1 | |
| | 1 | 12 | 8 | 6 | 5 | 4 | 3 | 3 | 3 | |
| | 2 | 15 | 11 | 8 | 7 | 5 | 4 | 4 | 4 | |
| | 3 | 18 | 13 | 10 | 9 | 7 | 6 | 5 | 5 | |
| | 4 | 21 | 15 | 12 | 10 | 8 | 7 | 6 | 6 | |
| 0.99 | 5 | 24 | 17 | 14 | 12 | 9 | 8 | 7 | 7 | |
| | 6 | 27 | 19 | 16 | 13 | 11 | 9 | 8 | 8 | |
| | 7 | 30 | 22 | 17 | 15 | 12 | 10 | 10 | 9 | |
| | 8 | 33 | 24 | 19 | 16 | 13 | 11 | | 10 | |
| | 9 10 | 30 39 | 20 28 | ∠1 22 | 18 | 14 | 13 | 12 | 11 | |
| | 9 10 | 36 38 | 26 28 | 21 22 | 18 19 | 14 15 | 13 14 | 12 | 11 | |

Table 1

Table 2. Operating characteristic function values for the sampling plan $(n, c = 2, t/\mu_0)$ with a given probability P^* for $\theta = 2$ in the Komal distribution

| P^* | с | t/μ_{o} | μ/μ_0 | | | | | | | |
|-------|----|-------------|-------------|---------|---------|---------|---------|---------|--|--|
| 1 | | -/0 | 2 | 4 | 6 | 8 | 10 | 12 | | |
| | 8 | 0.628 | 0.63903 | 0.90649 | 0.96427 | 0.98285 | 0.99050 | 0.99421 | | |
| | 6 | 0.942 | 0.60353 | 0.89286 | 0.95840 | 0.97986 | 0.98879 | 0.99314 | | |
| | 5 | 1.257 | 0.57027 | 0.87901 | 0.95227 | 0.97670 | 0.98697 | 0.99200 | | |
| 0.75 | 4 | 1.571 | 0.62485 | 0.89979 | 0.96121 | 0.98125 | 0.98958 | 0.99362 | | |
| 0.75 | 4 | 2.356 | 0.36642 | 0.77061 | 0.89984 | 0.94840 | 0.97016 | 0.98126 | | |
| | 3 | 3.141 | 0.50462 | 0.84243 | 0.93442 | 0.96704 | 0.98122 | 0.98832 | | |
| | 3 | 3.927 | 0.36400 | 0.75915 | 0.89207 | 0.94354 | 0.96702 | 0.97914 | | |
| | 3 | 4.712 | 0.25571 | 0.67185 | 0.84239 | 0.91432 | 0.94873 | 0.96703 | | |
| | 10 | 0.628 | 0.47896 | 0.83846 | 0.93391 | 0.96713 | 0.98140 | 0.98849 | | |
| | 7 | 0.942 | 0.48356 | 0.84045 | 0.93480 | 0.96759 | 0.98167 | 0.98866 | | |
| | 6 | 1.257 | 0.41666 | 0.80464 | 0.91756 | 0.95832 | 0.97618 | 0.98516 | | |
| 0.00 | 5 | 1.571 | 0.42487 | 0.80846 | 0.91931 | 0.95923 | 0.97671 | 0.98549 | | |
| 0.90 | 4 | 2.356 | 0.36642 | 0.77061 | 0.89984 | 0.94840 | 0.97016 | 0.98126 | | |
| | 4 | 3.141 | 0.19439 | 0.62503 | 0.81702 | 0.89986 | 0.93987 | 0.96124 | | |
| | 3 | 3.927 | 0.36400 | 0.75915 | 0.89207 | 0.94354 | 0.96702 | 0.97914 | | |
| | 3 | 4.712 | 0.25571 | 0.67185 | 0.84239 | 0.91432 | 0.94873 | 0.96703 | | |
| | 12 | 0.628 | 0.34358 | 0.76022 | 0.89524 | 0.94605 | 0.96882 | 0.98042 | | |
| | 8 | 0.942 | 0.37797 | 0.78223 | 0.90649 | 0.95228 | 0.97257 | 0.98285 | | |
| | 7 | 1.257 | 0.29278 | 0.72274 | 0.87518 | 0.93468 | 0.96187 | 0.97590 | | |
| 0.05 | 6 | 1.571 | 0.27164 | 0.70507 | 0.86532 | 0.92897 | 0.95833 | 0.97357 | | |
| 0.95 | 4 | 2.356 | 0.36642 | 0.77061 | 0.89984 | 0.94840 | 0.97016 | 0.98126 | | |
| | 4 | 3.141 | 0.19439 | 0.62503 | 0.81702 | 0.89986 | 0.93987 | 0.96124 | | |
| | 4 | 3.927 | 0.09683 | 0.48616 | 0.72245 | 0.83914 | 0.89982 | 0.93380 | | |
| | 4 | 4.712 | 0.04637 | 0.36642 | 0.62497 | 0.77061 | 0.85207 | 0.89984 | | |
| | 15 | 0.628 | 0.19626 | 0.63561 | 0.82519 | 0.90532 | 0.94353 | 0.96379 | | |
| | 11 | 0.942 | 0.16173 | 0.59552 | 0.80025 | 0.89008 | 0.93379 | 0.95724 | | |
| | 8 | 1.257 | 0.19955 | 0.63857 | 0.82689 | 0.90632 | 0.94416 | 0.96420 | | |
| 0.00 | 7 | 1.571 | 0.16597 | 0.59972 | 0.80271 | 0.89154 | 0.93470 | 0.95785 | | |
| 0.99 | 5 | 2.356 | 0.17690 | 0.60989 | 0.80854 | 0.89496 | 0.93683 | 0.95925 | | |
| | 4 | 3.141 | 0.19439 | 0.62503 | 0.81702 | 0.89986 | 0.93987 | 0.96124 | | |
| | 4 | 3.927 | 0.09683 | 0.48616 | 0.72245 | 0.83914 | 0.89982 | 0.93380 | | |
| | 4 | 4.712 | 0.04637 | 0.36642 | 0.62497 | 0.77061 | 0.85207 | 0.89984 | | |

| - | | |
|-----|------|--|
| Tah | le 2 | |
| | | |

Table3. Minimum ratio of μ/μ_0 for the acceptability of a lot with producer's risk of 0.05 for $\theta = 2$ in the Komal Distribution

| | r – | | | | | | | | | | |
|-------|-----|---------|-----------|---------|---------|---------|---------|---------|---------|--|--|
| D^* | C | | t/μ_0 | | | | | | | | |
| Г | | 0.628 | 0.942 | 1.257 | 1.571 | 2.356 | 3.141 | 3.927 | 4.712 | | |
| | 0 | 35.9896 | 35.9939 | 48.0300 | 30.0246 | 45.0273 | 60.0301 | 75.0519 | 90.0546 | | |
| | 1 | 7.7448 | 8.9970 | 8.4838 | 10.6030 | 9.1499 | 12.1986 | 15.2512 | 18.2998 | | |
| | 2 | 5.2314 | 5.5633 | 5.8858 | 5.4049 | 8.1057 | 6.7366 | 8.4224 | 10.1060 | | |
| | 3 | 3.7929 | 4.3183 | 3.9064 | 4.8823 | 5.5328 | 4.8450 | 6.0574 | 7.2682 | | |
| | 4 | 3.4050 | 3.6801 | 3.6230 | 3.7071 | 4.2912 | 3.9000 | 4.8759 | 5.8506 | | |
| 0.75 | 5 | 2.9130 | 2.9286 | 2.9270 | 3.0297 | 3.5663 | 3.3332 | 4.1673 | 5.0004 | | |
| | 6 | 2.7815 | 2.7397 | 2.8766 | 3.0990 | 3.0920 | 4.1222 | 3.6934 | 4.4317 | | |
| | 7 | 2.5222 | 2.5996 | 2.5001 | 2.7103 | 2.7571 | 3.6758 | 3.3527 | 4.0229 | | |
| | 8 | 2.3305 | 2.4911 | 2.5038 | 2.4247 | 2.5078 | 3.3434 | 3.0949 | 3.7136 | | |
| | 9 | 2.2982 | 2.2282 | 2.2583 | 2.5174 | 2.8256 | 3.0857 | 2.8924 | 3.4706 | | |
| | 10 | 2.1673 | 2.1780 | 2.2786 | 2.3103 | 2.6181 | 2.8796 | 2.7285 | 3.2740 | | |
| | 0 | 47.9833 | 53.9844 | 48.0300 | 60.0280 | 45.0273 | 60.0301 | 75.0519 | 90.0546 | | |
| | 1 | 11.2238 | 11.6172 | 12.0056 | 15.0046 | 15.9012 | 12.1986 | 15.2512 | 18.2998 | | |
| | 2 | 6.7459 | 6.7075 | 7.4236 | 7.3561 | 8.1057 | 10.8064 | 8.4224 | 10.1060 | | |
| | 3 | 5.1550 | 5.0053 | 5.7623 | 6.0485 | 5.5328 | 7.3762 | 6.0574 | 7.2682 | | |
| | 4 | 4.3501 | 4.1577 | 4.2698 | 4.5280 | 5.5594 | 5.7210 | 4.8759 | 5.8506 | | |
| 0.90 | 5 | 3.6269 | 3.6519 | 3.9079 | 3.6582 | 4.5436 | 4.7546 | 5.9444 | 5.0004 | | |
| | 6 | 3.3493 | 3.3157 | 3.2680 | 3.5951 | 3.8861 | 4.1222 | 5.1537 | 4.4317 | | |
| | / | 2.9915 | 3.0754 | 3.1488 | 3.1247 | 3.4264 | 3.6/58 | 4.5956 | 4.0229 | | |
| | 8 | 2.8614 | 2.8948 | 3.0527 | 3.1293 | 3.0869 | 3.3434 | 4.1800 | 3./136 | | |
| | 9 | 2.6429 | 2.7538 | 2.7370 | 2.8223 | 2.8250 | 3.0857 | 3.8578 | 3.4/00 | | |
| | 10 | 2.3/1/ | 2.0403 | 2.0980 | 2.3012 | 3.0469 | 2.8790 | 3.0002 | 3.2/40 | | |
| | 1 | 39.9770 | 14 2287 | 15 5010 | 15 0046 | 90.0229 | 21 1003 | 15 2512 | 90.0340 | | |
| | 2 | 8 2568 | 7 8471 | 8 9505 | 0 2781 | 8 1057 | 10 8064 | 13.5106 | 10.2998 | | |
| | 2 | 6.0602 | 5 6894 | 5 7623 | 6.0485 | 7 3218 | 7 3762 | 9 2221 | 7 2682 | | |
| | 4 | 4 6644 | 5 1076 | 4 9108 | 5 3364 | 5 5594 | 5 7210 | 7 1526 | 5 8506 | | |
| 0.95 | 5 | 4.1013 | 4.3696 | 4.3918 | 4.2747 | 4.5436 | 4.7546 | 5.9444 | 5.0004 | | |
| 0150 | 6 | 3.7268 | 3.8874 | 3.6558 | 4.0844 | 3.8861 | 4.1222 | 5.1537 | 6.1839 | | |
| | 7 | 3.4593 | 3.5479 | 3.4689 | 3.5323 | 4.0645 | 4.5680 | 4.5956 | 5.5143 | | |
| | 8 | 3.1260 | 3.0957 | 3.3241 | 3.4738 | 3.6362 | 4.1154 | 4.1800 | 5.0156 | | |
| | 9 | 2.9866 | 2.9278 | 2.9732 | 3.1231 | 3.3082 | 3.7671 | 3.8578 | 4.6290 | | |
| | 10 | 2.7733 | 2.7936 | 2.9062 | 2.8478 | 3.0489 | 3.4905 | 3.6002 | 4.3199 | | |
| | 0 | 95.9581 | 89.9655 | 96.0431 | 90.0314 | 90.0229 | 100 | 100 | 90.0546 | | |
| | 1 | 19.8959 | 19.4399 | 18.9867 | 19.3743 | 22.5021 | 21.1993 | 26.5042 | 31.8024 | | |
| | 2 | 10.5198 | 11.2525 | 10.4712 | 11.1863 | 11.0318 | 10.8064 | 13.5106 | 16.2113 | | |
| | 3 | 7.4158 | 7.7325 | 7.5919 | 8.3474 | 9.0709 | 9.7614 | 9.2221 | 11.0655 | | |
| | 4 | 5.9194 | 6.0533 | 6.1827 | 6.1375 | 6.7905 | 7.4118 | 7.1526 | 8.5824 | | |
| 0.99 | 5 | 5.0482 | 5.0840 | 5.3526 | 5.4889 | 5.4862 | 6.0575 | 5.9444 | 7.1326 | | |
| | 6 | 4.4804 | 4.4565 | 4.8064 | 4.5690 | 5.3916 | 5.1809 | 5.1537 | 6.1839 | | |
| | 7 | 4.0814 | 4.2530 | 4.1038 | 4.3354 | 4.6860 | 4.5680 | 5.7111 | 5.5143 | | |
| | 8 | 3.7858 | 3.8945 | 3.8629 | 3.8153 | 4.1695 | 4.1154 | 5.1452 | 5.0156 | | |
| | 9 | 3.5578 | 3.6199 | 3.6747 | 3.7160 | 3.7754 | 4.4105 | 4.7097 | 4.6290 | | |
| | 10 | 3.2762 | 3.4029 | 3.3185 | 3.3727 | 3.4647 | 4.0648 | 4.3639 | 4.3199 | | |

Table 3

5. Real Life Example

In this section, we have given a numerical application of the suggested acceptance sampling plan based on Komal distribution for the lifetime (in months) to first failure of 20 electric carts, (0.9, 1.5, 2.3, 3.2, 3.9, 5.0, 6.2, 7.5, 8.3, 10.4, 11.1, 12.6, 15.0, 16.3, 19.3, 2.6, 24.8, 31.5, 38.1, 53.0). Al-Omari et al. (2020) consider the data for acceptance sampling for Akash distribution. Below are reported and discussed the indexes adopted to evaluate the normality of the Komal distribution:

4.725 0.90 Min Q_1 Skewness 1.61 Q_3 Max 53 20.125 Kurtosis 0.96 52.1 Mean 14.68 Standard Deviation 13.66 Range Median 10.75 Variance 186.697 Standard Error 3.06

Table 4: The Descriptive Statistics for the First Failure of 20 Small Electric Carts

- The Kolmogorov-Smirnov (KS): it is mainly used in comparing two samples rather than one, with a standard distribution. KS is distribution free because the critical values do not depend on the particular distribution tested.
- The Cramér-von Misses (T), an alternative of KS test, is applied with cumulative distribution function.
- The Anderson-Darling (AD). It is a modification of Kolmogorov-Smirnov test because it gives more weight to the tails. It consider the specified distribution in calculating critical values. For this reason, this test is more sensitive but at the same time it needs to calculate critical values for each distribution) statistic.
- The Akaike information criterion (AIC), which estimates the quality of a statistical model for given data. It is based on information theory.
- The consistent Akaike information (CAIC).
- Bayesian information (BIC), used to choose the best model between different ones. Model with lowest BIC is considered the best.
- Hannan-Quinn information (HQIC), where:

$$KS = \underset{1 \le i \le n}{Max} \left(F(x_i) - \frac{i-1}{n}, \frac{1}{n} - F(x_i) \right)$$
$$AIC = 2k - 2MLL$$
$$CAIC = \frac{2kn}{n-k-1} - 2MLL$$
$$BIC = k \log(n) - 2MLL$$
$$HOIC = 2 \log \left\{ \log(n)(k - 2MLL) \right\}$$

Where k is the number of parameters and n is the sample size. MLL is the minimum value the log likelihood function for the estimated model. These values are summarized in Table 4.

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| Model | AIC | CAIC | BIC | HQIC | KS | -2MLL | p – value |
|-------|-------|-------|-------|-------|------|-------|-----------|
| Komal | 151.0 | 151.3 | 152.0 | 151.2 | 0.12 | 149.0 | 0.9 |
| Akash | 160.4 | 160.6 | 161.4 | 160.6 | 0.2 | 158.4 | 0.3 |

Table 5. The statistics AIC, CAIC, BIC, HQIC, KS, -2MLL and p-value for the Electric Carts Data

The better fit corresponds to the smaller values of KS, AIC, CAIC, BIC, HQIC and -MLL. Accordingly, using the KS test, it is found that the maximum distance between the data and the fitted of the Komal distribution is 0.1224 with p - value is 0.8905, which provides reasonable good fit for the first failure of 20 electric carts. The AIC value of Komal distribution is 151.0where AIC value of Akash distribution is 161.3. As Komal distribution shows minimum AIC value as compared to AIC value of Akash distribution, which shows that Komal distribution fits well than Akash distribution.

6. CONCLUSION

This research article presents a single-acceptance sampling plan for the Komal distribution, a newly introduced lifetime distribution model. The plan assumes the mean lifetime as a pre-specified quality characteristic. This methodology facilitates the assessment of both the producer's and consumer's risk based on the lifetime distribution of a representative sample. The manuscript includes various computations and numerical illustrations to validate the effectiveness of the proposed plan, and provides a comprehensive discussion of the outcomes. This study presents empirical data pertaining to the performance characteristics of a single-acceptance sampling methodology. The authors advise that both consumers and producers employ this sampling approach to decrease production costs and enhance operational efficiency. The research findings can inform the design of alternative acceptance sampling techniques, including group-based and double-sampling approaches, for the Komal distribution as well as other relevant probability distributions.

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